

Fiscal and Monetary Interactions

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- 1 – The Fiscal Theory of the Price Level (FTPL)
 - 2 – Ricardian *versus* non-Ricardian regimes
 - 3 – The monetarist explanation of the price level
 - 4 – The FTPL explanation of the price level
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 - 6 – FTPL in a two-country monetary union
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- References

- The (FTPL) was initially made popular by Leeper (1991), Sims (1994) and Woodford (1994, 1995).
- One can trace the discussion back to Sargent and Wallace (1975), and to the controversy of using rules to control the nominal interest rate, which may lead to price level indeterminacy.
- Leeper-Sims-Woodford argue that it will be then up to the government budget constraint to play a key role in the determination of the price level.

Main idea:

- Fiscal policy may have a relevant role, at least as important as monetary policy, in determining the price level.

In a nutshell:

- Direct effects of fiscal policy on the price level.
- The price level is determined via the intertemporal government budget constraint.
- The price level (P) adjusts to ensure that the current real value of outstanding government debt (B) is equal to the actual real value of future primary budget balances.

$$B / P = \text{present value of future primary budget balances}$$

Discussions and critics: McCallum (1999a, 1999b, 2001), Buiter (2002).

Empirical assessments: Canzoneri, Cumby and Diba (1996, 2000), Cochrane (1998) and Woodford (1995), Afonso (2008).

- **Ricardian regime**, the fiscal authority fully finances new government debt via future tax revenues.
- **Non-Ricardian regime**, the fiscal authority does not commit to fully finance debt via future taxes and monetary financing can then occur.
- Ricardian regime:
 - primary government budget balances react to government debt to ensure fiscal solvency.
 - Money and prices are determined by money supply and demand [*active monetary policy*]. This is linked to the idea of Ricardian Equivalence, where budget deficits do not affect income and interest rates.

- Non-Ricardian regime:
 - Primary budget balances can be determined by the government without taking into account the level of government debt.
 - Money and prices would then need to adjust to the level of government debt to guarantee the fulfilment of the government intertemporal budget constraint [*passive monetary policy*].

Sargent e Wallace (1981):

- regime of monetary dominance = Ricardian regime
- regime of fiscal dominance = non-Ricardian regime

Buiter (1999):

- Ricardian fiscal rule = the government intertemporal budget constraint needs to be fulfilled for all values of the endogenous variables;
- Non-Ricardian fiscal rule = the government intertemporal budget constraint needs to be fulfilled only for the equilibrium values of the endogenous variables.

	Terminology		
Leeper (1991)	- Passive fiscal policy - Active monetary policy	<i>vs.</i>	- Active fiscal policy - Passive monetary policy
Woodford (1995)	Ricardian regime	<i>vs.</i>	Non-Ricardian regime
Canzoneri and Diba (1996)	Monetary dominance	<i>vs.</i>	Fiscal dominance
Buiter (1999)	Ricardian fiscal rule	<i>vs.</i>	Non-Ricardian fiscal rule

From the quantity theory of money, the money demand equation is

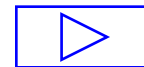
$$M_t v_t = P_t y_t \quad (1)$$

M – nominal money stock;

v – velocity circulation of money;

P – price level;

y – real income.



- In a Ricardian regime, the monetary authority determines the stock of money and the price level via (1).
- The fiscal authority has to achieve the necessary primary budgetary surpluses in order that its budget constraint is consistent with the price level from (1).

- Non-Ricardian regime: the fiscal authority may autonomously decide on the budget balance and government debt, influencing the determination of the price level.
- The monetary authority sets endogenously the money supply from

$$M_t = (P_t y_t) / v_t$$

and takes P from the government budget constraint.

- Unpleasant monetarist arithmetic: inflation is still a monetary result.
- FTPL is less orthodox, price level would be determined via the fiscal behaviour regardless of changes in the money stock.

Carlstrom e Fuerst (2000):

- “weak form” of FTPL: Sargent and Wallace (1981), fiscal policy is exogenous, impinges on the price level via the money supply;
- “strong form” of FTPL: Leeper-Sims-Woodford, fiscal policy affects the price level independently of the money supply.

Orthodox view of the price level (McCallum, 1989, 1999b),

Money demand:

$$m_t - p_t = a_0 + a_1 y_t + a_2 i_t + u_t \quad (2)$$

$$a_1 > 0; a_2 < 0;$$

m – logarithm of money stock;

p – logarithm of the price level;

y – logarithm of income;

i – one period nominal interest rate (logarithm);

u – white noise ($E_t u_t = 0$).

$$i_t = r_t + \Delta p_{t+1}^e \quad (3)$$

r – real interest rate;

Δp_{t+1}^e – price level expectations.

With rational expectations, the expected inflation is

$$E_t \Delta p_{t+1} = E_t (p_{t+1} - p_t) = E_t p_{t+1} - p_t \quad (4)$$

$$m_t - p_t = \gamma + \alpha (E_t p_{t+1} - p_t) + u_t \quad (5)$$

$$\alpha \equiv a_2; \gamma \equiv a_0 + a_1 y + a_2 r$$

recall:

$$m_t - p_t = a_0 + a_1 y_t + a_2 i_t + u_t$$

Money stock follows

$$m_t = m_{t-1} + \mu \quad (6)$$

Process for the price level

$$p_t = \phi_0 + \phi_1 m_{t-1} + \phi_2 u_t \quad (7)$$

$$p_{t+1} = \phi_0 + \phi_1 m_t + \phi_2 u_{t+1} \quad (8)$$

Taking the expected value of (8) and using (6),

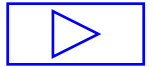
$$E_t p_{t+1} = \phi_0 + \phi_1 (m_{t-1} + \mu) \quad (10)$$

Substituting (6) in (5), money demand becomes

$$m_{t-1} + \mu = \gamma + \alpha E_t p_{t+1} + (1 - \alpha) p_t + u_t \quad (12)$$

and using the expected price (10) and the price process (7),

$$m_{t-1} + \mu = \gamma + \alpha [\phi_0 + \phi_1 (m_{t-1} + \mu)] + (1 - \alpha) [\phi_0 + \phi_1 m_{t-1} + \phi_2 u_t] + u_t \quad (13)$$



Solution for p (with the method of indeterminate coefficients),

$$p_t = m_t - (\gamma + \alpha\mu) - \frac{1}{(1-\alpha)} u_t \quad (24)$$

- prices grow on a one-to-one basis with money,
- Prices respond positively to negative shocks in money demand ($u_t < 0$).

- The FTPL contests the idea that “Inflation is always and everywhere a monetary phenomenon” [Milton Friedman]
- From the money demand equation (1)=(27)

$$M_t v_t = P_t y_t \quad (27)$$

- Assuming velocity depends on the nominal interest rate $v_t = v(i_t)$

$$M_t (i_t)^b = P_t y_t \quad (28)$$

$$b > 0,$$

- Using logs, real interest rate, r , with perfect foresight

$$\ln M_t = \ln P_t + \ln y_t - b[\ln r_t + \ln P_{t+1} - \ln P_t] \quad (29)$$

Again with constant income and real interest rate, and fixed money supply, the price level is given by the difference equation

$$\ln P_{t+1} - \ln(M / y) = \frac{1+b}{b} [\ln P_t - \ln(M / y)] - \ln r \quad (31)$$

Depending of the initial value for the price level, there are infinite trajectories for such difference equation.

Assuming an initial value for P (not resulting from theory or from optimization of the money demand function, critic of Woodford) ensures that prices do not follow an explosive path.

$$\ln P_0 = \ln(M / y) + \ln r \quad (32)$$

Leeper-Sims-Woodford: in a non-Ricardian regime, the government budget constraint determines a unique price level.

$$\frac{B_t}{P_t} = \sum_{s=0}^{\infty} \frac{s_{t+s}}{(1+r)^{s+1}} \quad (33)$$

B_t – nominal government liabilities (including debt and money base);
 s_t – real primary government budget surplus (with seigniorage revenue);
 r – real interest rate, constant by hypothesis;

Transversality condition (no-Ponzi game condition):

$$\lim_{s \rightarrow \infty} \frac{B_{t+s}}{(1+r)^{s+1}} = 0 \quad (34)$$

$$\frac{B_t}{P_t} = \sum_{s=0}^{\infty} \frac{S_{t+s}}{(1+r)^{s+1}} \quad (33)$$

- In a non-Ricardian regime, (33) is fulfilled if after the government has chosen a sequence for primary balances, the price level adjusts endogenously.
- If (33) is fulfilled for any price level, then it will be fiscal policy to adjust implying a Ricardian regime.

Infinitely lived households, maximizing an additive utility function that includes money [McCallum (1999a)]

$$U(c_t, m_t) = (1 - \sigma)^{-1} A_1 c_t^{1-\sigma} + (1 - \eta)^{-1} A_2 m_t^{1-\eta} \quad (35)$$

$$\sigma > 0; \eta > 0;$$

A – productivity (TFP);

c_t – real consumption;

M – nominal stock of money;

$$m_t = M_t / P_t,$$

Household budget constraint in nominal terms

$$P_t(y - tx_t) = P_t c_t + M_{t+1} - M_t + \frac{B_{t+1}}{1+i_t} - B_t \quad (36)$$

y – output, assumed constant;

tx_t – (lump-sum) taxes;

B_t – one period government debt;

i – nominal interest rate.

Household budget constraint in real terms, using $bt = Bt/Pt$, multiply both sides by (Pt/P_{t+1}) ,

$$(y - tx_t) \frac{P_t}{P_{t+1}} = \frac{P_t}{P_{t+1}} c_t + m_{t+1} - \frac{P_t}{P_{t+1}} m_t + \frac{1}{1+i_t} b_{t+1} - \frac{P_t}{P_{t+1}} b_t \quad (38)$$

Household optimization problem

$$\left\{ \begin{array}{l} \text{Max } U(c_t, m_t) = (1 - \sigma)^{-1} A_1 c_t^{1-\sigma} + (1 - \eta)^{-1} A_2 m_t^{1-\eta} \\ \text{s. a} \quad (y - tx_t) \frac{P_t}{P_{t+1}} = \frac{P_t}{P_{t+1}} c_t + m_{t+1} - \frac{P_t}{P_{t+1}} m_t + \frac{1}{1+i_t} b_{t+1} - \frac{P_t}{P_{t+1}} b_t \end{array} \right. \quad (39)$$

1st order condition (assuming $P_{t+1}^e = P_{t+1}$), usual Euler equation,

$$U_c(c_t, m_t) = \frac{P_t}{P_{t+1}} \frac{1+i_{t+1}}{1+r} U_c(c_{t+1}, m_{t+1}) \quad (40)$$

Consolidated government budget constraint (with fiscal and monetary authorities)

$$P_t(g_t - tx_t) = M_{t+1} - M_t + \frac{B_{t+1}}{1+i_t} - B_t \quad (41)$$

B_{t+1} – debt issued in t , at price $1/(1+i_t)$, reimbursed in period $t+1$;

g_t – real government spending;

tx_t – real government revenue.

Government budget constraint in real terms

$$g_t - tx_t = \frac{M_{t+1} - M_t}{P_t} + \frac{B_{t+1}}{1+i_t} \frac{1}{P_t} - \frac{B_t}{P_t} \quad (43)$$

- Using $bt = Bt/Pt$; $mt = Mt/Pt$,
- Real interest rate, r , via the Fisher equation

$$1 + i_t = (1 + r_t)(1 + \pi_{t+1}^e) \quad (46)$$

- Perfect inflation foresight

$$\pi_{t+1}^e = \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \quad (47)$$

Recalling the above assumed constant money base, and after some algebraic manipulation

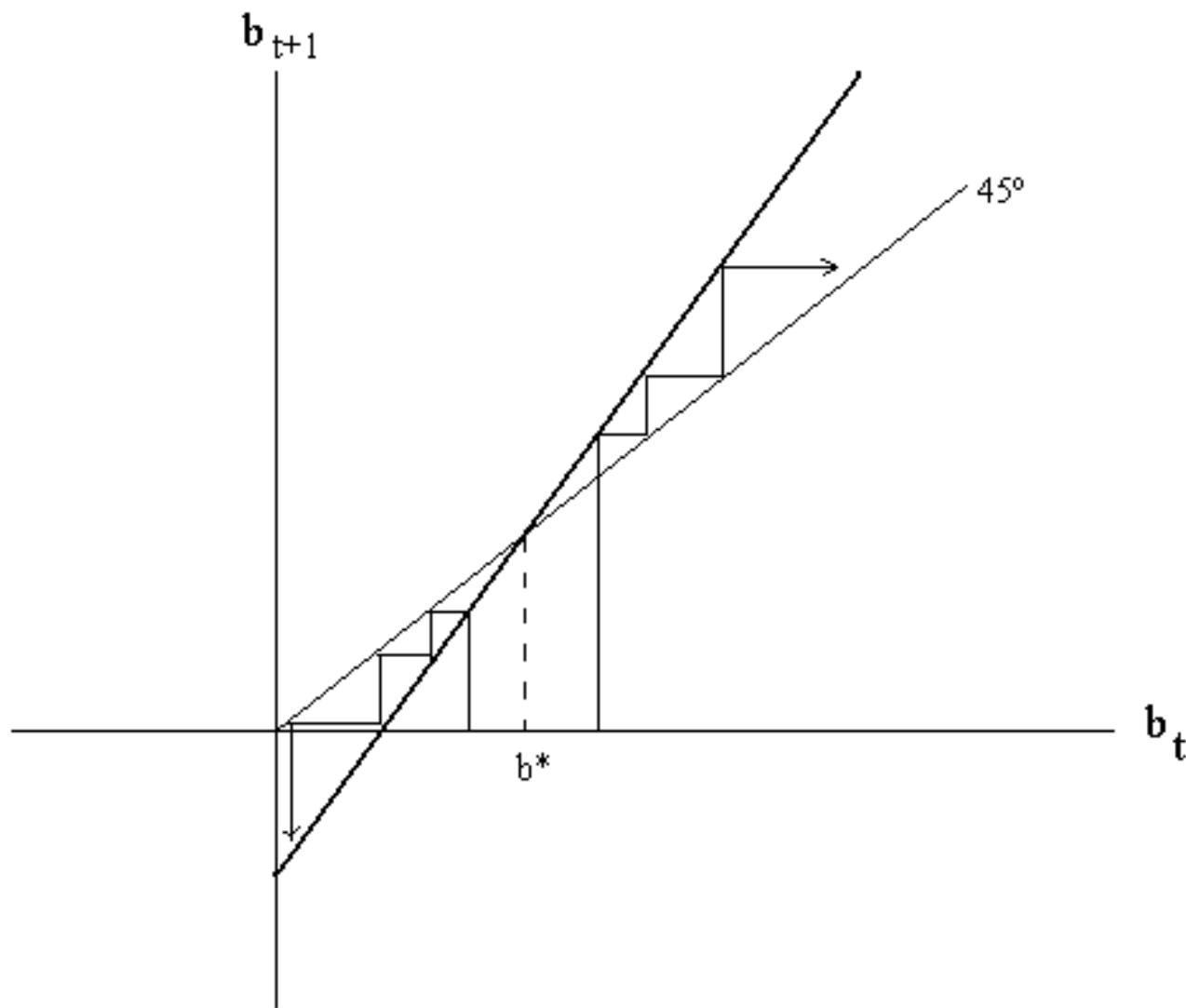
The government budget constraint, with a constant real interest rate, keeping the budget balance also constant $(g_t - tx_t) = (g - tx)$

$$b_{t+1} = (1 + r)b_t + (1 + r)(g - tx) \quad (53)$$

...more succinctly, with $s = tx - g$

$$b_{t+1} = (1 + r)(b_t - s) \quad (54)$$

$b_t = B_t/P_t$ will follow an explosive path since $(1+r) > 0$.



Government
budget constraint
and debt path

Starting from b_t in the x-axis (abscissa), moving vertically to the budget constraint (bold) line, then horizontally to the 45 degree line, again vertically to the budget constraint line, is obvious the explosive path of government debt.

- With initial value $b < b^*$, b diverges towards negative values, not consistent with the non-negativity restriction for the stock of government debt.
- With $b > b^*$, b also diverges, growing without bound.
- From (54), the growth rate of b is

$$\frac{b_{t+1}}{b_t} = (1+r)\left(1 - \frac{s}{b_t}\right) \quad (55)$$

converging to $(1+r)$ as b increases.

- In this case the government is doing Ponzi games, and does not satisfy the transversality condition (34).

To avoid an explosive path for the debt, initially, $b_0=b^*$, ensuring that b remains constant.

$$b_0 = -(1+r)(g - tx) / r \quad (56)$$

Such initial value for b gives

$$\frac{B_0}{P_0} = \left(\frac{1+r}{r} \right) (tx - g) \quad (58)$$

and P_0 is determined by

$$P_0 = rB_0 / [(1+r)(tx - g)] \quad (59)$$

In other words, the price level depends directly of the ratio between the initial debt level and the budget balance.

In a monetary union, a single monetary policy may not suffice to control the price level. If one country has a non-Ricardian fiscal policy, such behaviour will impinge on the inflation rate of the union.

Assume:

- monetary union with 2 countries;
- one single Central Bank;
- government debt in each country is denominated in the single currency;

B_{1t}^T, B_{2t}^T – total government debt issued by countries 1 and 2.

Difference equation for total government debt of countries 1 and 2

$$B_{1t+1}^T = (1 + i_t) \left(B_{1t}^T + P_t (g_{1t} - tx_{1t}) \right) \quad (60)$$

$$B_{2t+1}^T = (1 + i_t) \left(B_{2t}^T + P_t (g_{2t} - tx_{2t}) \right) \quad (61)$$

i – nominal interest rate, equal for the 2 countries;

P – price level, equal for the 2 countries;

g_{it} – primary government spending for country i ($i=1, 2$);

tx_{it} government revenue for country i ($i=1, 2$).

- Assume the Central Bank holds similar amounts of government debt of each country,

$$B_{1t}^C = B_{2t}^C$$

$$B_{1t+1}^C = (1 + i_t) \left(B_{1t}^C + 0.5(M_{t+1} - M_t) \right) \quad (62)$$

$$B_{2t+1}^C = (1 + i_t) \left(B_{2t}^C + 0.5(M_{t+1} - M_t) \right) \quad (63)$$

Government debt held by the public in countries 1 and 2

$$B_{1t} = B_{1t}^T - B_{1t}^C \quad B_{2t} = B_{2t}^T - B_{2t}^C$$

$$B_{1t+1} = (1 + i_t) \left(B_{1t} + P_t (g_{1t} - tx_{1t}) - 0.5(M_{t+1} - M_t) \right) \quad (64)$$

$$B_{2t+1} = (1 + i_t) \left(B_{2t} + P_t (g_{2t} - tx_{2t}) - 0.5(M_{t+1} - M_t) \right) \quad (65)$$

Aggregating the 2 government budget constraints, the development for government debt in the monetary union is $B_t^U \equiv B_{1t} + B_{2t}$

$$B_{t+1}^U = (1 + i_t) \left(B_t^U + P_t [(g_{1t} - tx_{1t}) + (g_{2t} - tx_{2t})] - (M_{t+1} - M_t) \right) \quad (66)$$

Implications (1)

- When both countries follow a Ricardian fiscal policy, government debt developments do not impinge on the price level of the monetary union.
- If one country follows a non-Ricardian fiscal policy, and the other sticks to a Ricardian behaviour, there will be an effect on the union price level.

Implications (2)

- To ensure the stability of the price level in a monetary union, the union consolidated government budget constraint would have to be fulfilled.
- The sum of the actual value of the future budget surpluses in the two countries will need to match the real value of the outstanding stock of government debt in the monetary union.
- When the actual value of the budget surpluses of one country decreases, the actual value of the budget surpluses of the other country has to increase.
- For instance, fiscal rules can help in ensuring a country by country Ricardian behaviour of the fiscal authorities.

Canzoneri, Cumby and Diba (2000) use a bi-variate VAR for the U.S. (1951-1995)

$$s_t = a_{10} + \sum_{i=0}^n a_{1i} s_{t-i} + \sum_{i=0}^n b_{1i} w_{t-i} \quad (67)$$

$$w_t = a_{20} + \sum_{i=0}^n a_{2i} s_{t-i} + \sum_{i=0}^n b_{2i} w_{t-i} \quad (68)$$

s – real primary budgetary surplus, % of GDP

w – real government responsibilities, % of GDP, including government debt and monetary base.

- Ricardian regime, $a_{2i} < 0$, increase in budgetary surpluses is used to pay back debt;

- non-Ricardian regime, $a_{2i} > 0$.

- Ricardian regime, $b_{1i} > 0$, the government increases the budgetary surpluses to face higher indebtedness;
- non-Ricardian regime, $b_{1i} = 0$, budgetary surpluses do not react to increases in government indebtedness.
- Canzoneri, Cumby and Diba (2000) conclude for $a_{2i} < 0$, a Ricardian regime, therefore little evidence for the FTPL.

Afonso (2008): panel approach for the EU-15 (1970-2003),

$$\Delta s_{it} = \delta \Delta s_{it-1} + \theta \Delta b_{it-1} + \lambda_s \Delta z_{it-1} + \Delta u_{it} \quad (69)$$

s – primary balance, % of GDP;

b – debt-to-GDP ratio;

z – output gap, difference between actual GDP and potential GDP, % of potential GDP;

u_{it} – independent across countries.

i) $\theta = 0$, primary balance does not react to the level of public debt, a non-Ricardian fiscal regime;

ii) $\theta > 0$, the government tries to increase the primary balance in order to react to the existing stock of public debt, which could be seen as a sign of a Ricardian fiscal regime.

Alternatively,

$$\Delta b_{it} = \gamma \Delta s_{it-1} + \varphi \Delta b_{it-1} + \lambda_b \Delta z_{it-1} + \Delta v_{it} \quad (70)$$

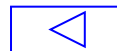
- i) a Ricardian fiscal regime is not rejected when $\gamma < 0$, as most likely the government is using budget surpluses to reduce outstanding government debt.
- ii) With $\gamma \geq 0$ there may be a non-Ricardian regime, a regime of fiscal dominance.

The results give support to the Ricardian fiscal regime hypothesis throughout the sample period.

- Afonso, A. (2008). “Ricardian Fiscal Regimes in the European Union”, *Empirica* 35 (3), 313–334.
- Buiter, W. (2002). “The Fiscal Theory of The Price Level: A Critique,” *Economic Journal*, 112 (481), 459-480.
- Canzoneri, M.; and Diba, B. (1996). “Fiscal Constraints on Central Bank Independence and Price Stability,” CEPR Discussion Paper 1463.
- Canzoneri, M.; Cumby, R and Diba, B. (2000). “Is the Price Level Determined by the Needs of Fiscal Solvency?” *American Economic Review*.
- Carlstrom, C. and Fuerst, T. (2000). "The fiscal theory of the price level," Federal Reserve Bank of Cleveland. *Economic Review*, 1st quarter, 22-32.
- Christiano, L. and Fitzgerald, T. (2000). "Understanding the fiscal theory of price level," Federal Reserve Bank of Cleveland. *Economic Review*, 2nd quarter, 2-38. [*] <http://www.clevelandfed.org/research/Review/2000/IIq.pdf>
- Cochrane, J. (1998). “Long-term Debt and Optimal Policy in the Fiscal Theory of the Price Level,” NBER Working Paper 6771.
- Leeper, E. (1991). “Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies,” *Journal of Monetary Economics*, 27 (1), 129-147.
- McCallum, B. (1999a). “Issues in the Design of Monetary Policy Rules,” in Taylor, J. and Woodford, M. (eds.), *Handbook of Macroeconomics*, North-Holland Pub. Co. [*]

- McCallum, B. (1999a). “Issues in the Design of Monetary Policy Rules,” in Taylor, J. and Woodford, M. (eds.), *Handbook of Macroeconomics*, North-Holland Pub. Co. [*]
- McCallum, B. (1999b). “Theoretical Issues Pertaining to Monetary Unions,” NBER Working Paper 7393. [*]
- McCallum, B. (2001). “Indeterminacy, Bubbles, and the Fiscal Theory of Price Level Determination,” *Journal of Monetary Economics*, 47 (1), 19-30.
- Sargent, T. and Wallace, N. (1975). “‘Rational’ expectations, the optimal monetary instrument and the optimal money supply rule”, *Journal of Political Economy*, 83 (2), 241-254.
- Sargent, T. and Wallace, N. (1981). “Some Unpleasant Monetarist Arithmetic”, Federal Reserve Bank of Minneapolis *Quarterly Review*, 5, 1-17.
<http://www.minneapolisfed.org/research/QR/QR531.pdf>
- Sims, C. (1994). “A Simple Model for the Study on the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory*, 4 (3), 381-399.
- Woodford, M. (1994). “Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy,” *Economic Theory*, 4 (3), 345-380.
- Woodford, M. (1995). “Price Level Determinacy without Control of Monetary Aggregate,” *Carnegie-Rochester Conference Series on Public Policy*, 43 (1), 1-46.

Quantity theory of money underpinning



- Equation of exchange (Fisher, 1911)

$$MV = PT$$

- GDP version

$$MV = PY$$

- taking natural logarithms and differentiating with respect to time,

$$\frac{d(\ln M)}{dt} + \frac{d(\ln V)}{dt} = \frac{d(\ln P)}{dt} + \frac{d(\ln Y)}{dt}$$

$$\frac{1}{M} \frac{dM}{dt} + \frac{1}{V} \frac{dV}{dt} = \frac{1}{P} \frac{dP}{dt} + \frac{1}{Y} \frac{dY}{dt}$$

$$\dot{m} + \dot{v} = \dot{p} + \dot{y} \quad \rightarrow \quad \dot{m} = \pi + \dot{y} - \dot{v}$$

M – money hold by the public;

V – velocity circulation of money;

P – price level;

T – economic transactions;

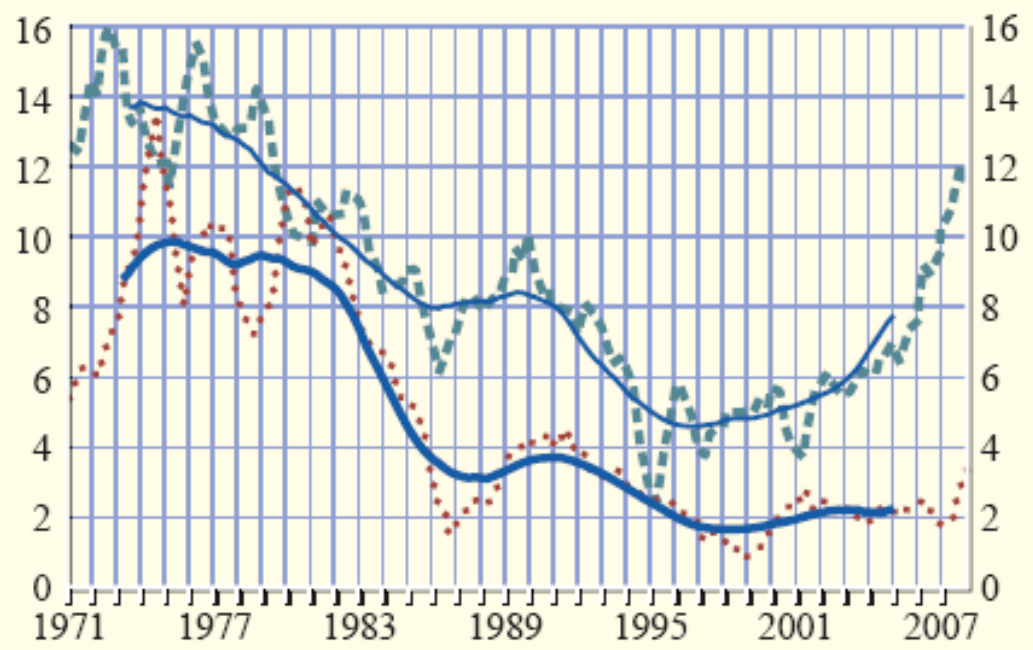
Y – GDP.

$$\dot{m} = 1.5\% + 2.25\% - (-0.75\%) = 4.5\%$$

Chart I Relationship between the HICP and M3 in the euro area

(annual rates of growth, low-frequency component reflects periodicity > 10 years)

- HICP low-frequency component
- HICP
- - - M3 corrected for the estimated impact of portfolio shifts
- M3 low-frequency component



Source: ECB (2008a).

Notes: Low-frequency components derived from a symmetric fixed-length Christiano-Fitzgerald band-pass filter applied to the annualised quarter-on-quarter rates of growth. The peak of money growth leads inflation by eight to twelve quarters.



The solution for p can be found using the method of indeterminate coefficients (equating the coefficients for m_{t-1} , u_t , and for the constant in both sides of (13)):

$$m_{t-1} + \mu = \gamma + \alpha[\phi_0 + \phi_1(m_{t-1} + \mu)] + (1 - \alpha)[\phi_0 + \phi_1 m_{t-1} + \phi_2 u_t] + u_t \quad (13)$$

$$1 = \alpha\phi_1 + (1 - \alpha)\phi_1, \quad (14)$$

$$0 = (1 - \alpha)\phi_2 + 1, \quad (15)$$

$$\mu = \gamma + \alpha\phi_1\mu + (1 - \alpha)\phi_0 + \alpha\phi_0, \quad (16)$$

The solution of (14)-(16) is:

$$\phi_0 = \mu - \gamma - \alpha\mu, \quad (17)$$

$$\phi_1 = 1, \quad (18)$$

$$\phi_2 = -1/(1 - \alpha). \quad (19)$$



Substituting (17)-(19) in the price equation (7),

$$p_t = \mu - \gamma - \alpha\mu + m_{t-1} - \frac{1}{(1-\alpha)}u_t \quad (20)$$

$$p_t = (1-\alpha)\mu - \gamma + m_{t-1} - \frac{1}{(1-\alpha)}u_t \quad (21)$$

using the money stock process (6),

$$p_t = (1-\alpha)\mu - \gamma + m_t - \mu - \frac{1}{(1-\alpha)}u_t \quad (22)$$

$$p_t = m_t - \gamma - \alpha\mu - \frac{1}{(1-\alpha)}u_t \quad (23)$$

$$p_t = m_t - (\gamma + \alpha\mu) - \frac{1}{(1-\alpha)}u_t \quad (24)$$